EEN 360
MODERN
CONTROL

FALL 2008
TEST 2
50 MINUTES

NDU
ECCE DEPARTMENT
S. C. ELMURR

NAME:----------

NOTE1: OPEN BOOK, OPEN NOTES, CLOSED OLD TESTS AND SOLUTIONS.
NOTE2: SHOW ALL WORK IN ORDER TO RECEIVE FULL CREDIT.

1. The open- loop transfer function of a single-loop negative feedback system is

$$GH(s) = \frac{K(s+8)}{s(s+3)(s+5)(s+7)(s+15)}$$

This system is called conditionally stable because it is stable for only a range of the gain K as follows: $k_1 < K < k_2$. Using the Routh-Hurwitz criteria and the root-locus method, determine the range of the gain for which the system is stable. Sketch the root locus for $-\infty < K < \infty$.

Hint:(the breakaway points are: -11.6, -9.34, -6.4, -4.03, -1.12. and $k_2 = 1010$.

$$GH(s) = \frac{K(s+8)}{s(s+3)(s+5)(s+7)(s+15)}$$

n poles : 5 $\quad 0, -3, -5, -7, -15$

m zero : 1 $\quad -8 \quad$ 4 at $\pm\infty$

Asymptote $\quad 2|n-m| = 2|5-1| = 2|4|$

Centroid $\quad \frac{\Sigma p - \Sigma z}{|n-m|} = \frac{0-3-5-7-15-(-8)}{5-1} = \frac{-32+8}{4} = \frac{-22}{4}$

Centroid $= -5.5$

$x = 0, 1, 2, 3$

$K>0$ RL >0 $\quad \frac{(2\pi+1)\pi}{|n-m|} \Rightarrow 45, 135, 225, 315$

$\frac{(2(0)+1)\pi}{4} = \frac{\pi}{4} \quad \frac{(2(1)+1)\pi}{4} = \frac{3\pi}{4} \quad \frac{(2(2)+1)\pi}{4} = \frac{5\pi}{4}$

$\frac{(2(3)+1)\pi}{4} \quad \frac{7\pi}{4}$

$K<0$ CRL $\quad \frac{2\pi\pi}{n-m} \Rightarrow 0, 90, 180, 270$

$\frac{2(0)\pi}{4} = 0 \quad \frac{2\pi}{4} = \frac{\pi}{2} \quad \frac{2(2)\pi}{4} = \pi \quad \frac{2(3)\pi}{4} = \frac{6}{4}$

NOTRE DAME UNIVERSITY
Zouk Mosbeh - Lebanon

Name:
Course N
Section:
Date:

$$GH(s) = \frac{K(s+8)}{s(s+3)(s+5)(s+7)(s+15)}$$

n poles 5 $\quad 0, -3, -5, -7, -15$

m zero 1 $\quad -8$ $\quad 4$ at $\pm\infty$

Asymptote $\quad 2|n-m| = 2|5-1| = 2|4|$

Centroid $\quad \frac{\Sigma_p - \Sigma_z}{|n-m|} = \frac{0-3-5-7-15-(-8)}{5-1} = \frac{-30+8}{4} = \frac{-22}{4}$

Centroid $= -5.5$

$x = 0, 1, 2, 3$

$K>0$ RL >0 $\quad \frac{(2x+1)\pi}{|n-m|} \Rightarrow 45, 135, 225, 315$

$\frac{(2(0)+1)\pi}{4} = \frac{\pi}{4}$ $\quad \frac{(2(1)+1)\pi}{4} = \frac{3\pi}{4}$ $\quad \frac{(2(2)+1)\pi}{4} = \frac{5\pi}{4}$

$\frac{(2(3)+1)\pi}{4} = \frac{7\pi}{4}$

$K<0$ CRL $\quad \frac{2x\pi}{n-m} \Rightarrow 0, 90, 180, 270$

$\frac{2(0)\pi}{4} = 0$ $\quad \frac{2\pi}{4} = \frac{\pi}{2}$ $\quad \frac{2(2)\pi}{4} = \pi$ $\quad \frac{2(3)\pi}{4} = \frac{6\pi}{4}$

$K(s+8)+s(s+3)(s+5)(s+7)(s+15)=0$

$(s^2+3s)(s+5)(s+7)(s+15)$

$s^3+5s^2+3s^2+15s)(s+7)(s+15)=0$

$(s^4+8s^3+15s^2+7s^3+56s^2+105s)(s+15)$

$s^5+15s^4+71s^3+105s^2+15s^4+225s^3+1065s^2+1575s$

$s^5+30s^4+296s^3+1170s^2+(1575+k)s+8k$

s^5	1	296	$1575+K$
s^4	30	1170	$8K$
s^3	254	$1575-1,0239K$	0
s^2	$986.14+0.119K$	$8k$	0
s^1	$\frac{1553170.5-822.15K-0.122K^2}{986.14+0.119K}$	0	0
s^0	K		

$(986.14+0.119)s^2+8K$

We replace

for $K_2 = 1010$

[redacted]

[redacted]

$$(986.14 + 0.119)s^2 + 8080$$

$$986.14s^2 + 0.119s^2 + 8080 = 0$$

$$986.259s^2 + 8080 = 0$$

$$986.259s^2 = -8080$$

$$s^2 = \frac{-8080}{986.256}$$

$$s^2 = -8.19$$

$$s^2 = j8.19$$

$$s = \pm\sqrt{2.86}$$

for $K = 0$